Summary

Multiple factor models of security covariance have been widely adopted by investment practitioners as a means to forecast the volatility of portfolios. Most of these models assume single period assumption, where future risk levels are presumed not to vary over time. Usually there are three types of multiple factor covariance models that are popular. The first is the exogenous factor model, where the common factors are typically macroeconomic state variables such as interest rates, levels of production, inflation and energy costs. Macroeconomic factor models have a limitation in the sense that they cannot readily capture risks that are not part of the economic state. The second type of factor models are based on covariance matrix that uses security characteristics as proxies for factors of commonality. The limitation with these models is that there are often so many overlapping effects that it is nearly impossible to correctly sort them all out. The third type of model in use today is the so called blind factor model. In such models, the factors are not specified as being any measurable real world phenomena, but rather both the factors and the β's to those factors are inferred from the security behavior themselves. Thus the factors evolve over time.

All the above models make an assumption of a single future period. No provision is made for forecast levels of portfolio volatility to vary through time.

The news information can be segregated in to two segments.The fist is news, that are wholly unanticipated by market participants. The second mechanism is announcements, information arrivals that are anticipated with respect to time but not with respect to content. How do participants adjust to news and announcements ? There are many studies that have considered the usage of option implied volatility as a mechanism for predicting future volatility levels. Many papers have shown that implied volatility is a very efficient predictor and historical volatility adds very little predictive power to models that already utilize option implied volatility

The key issue is the way to incorporate GARCH or implied volatility models in to portfolio risk computations. Once we discard the assumption of random walk,one cannot scale the variances across time. One of the ways to annualize high frequency metrics by explicit time series process embodied in GARCH, but we cannot simultaneously assume a GARCH process for estimating risk forecast and assume a random walk process for rescaling that estimate to annual units.

The problem with GARCH approach is that processes are designed to model the impact of a shock on system that is already close to an equilibrium condition. The basic problem with using GARCH process for estimating announcement induced volatility is that there is always a period before the announcement when the vol is low and GARCH processes will adjust the conditional volatility estimate downward relative to the long-term mean. On the date of announcement when the volatility spikes, the GARCH model still takes a while to catch up and when it does catch up, the market vol would have already come down. The paper suggests that it is better to use implied vol rather than GARCH.

The standard formulation of portfolio risk is

$$V_p = \sum_{i=1}^{n}\sum_{j=1}^{n} e_{p,i}e_{p,j}\sigma_{f(i)}\sigma_{f(j)}\rho_{i,j} + \sum_{i=1}^{m} w_k^2\sigma_{s(k)}^2$$

where

$$e_{pi} = \sum_{i=1}^{m} w_k \beta_{k,i}$$

V_p is the variance of portfolio return, n is the number of factors in the risk model, m is the number of securities in the portfolio, $e_{p,i}$ is the exposure to factor i, $\sigma_{f(i)}$ is the standard deviation returns attributed to factor i, $\rho_{i,j}$ is the correlation between returns to factor i and factor j , w_k is the weight of security k in the portfolio and $\sigma_{s(k)}$ is the standard deviation of security returns for security k and $\beta_{k,j}$ is the beta of the security k to factor i.

Usually implied volatilities are an upward biased estimator of expected volatility, and hence the paper makes use of changes in the implied volatility levels rather than the actual implied volatility numbers.

Denote V_k as conditional estimate of the future volatility of security k, as normally V_k times an adjustment factor M_k

$$V_k^* = V_k \times M_k$$

where

$$M_{k,t} = \frac{I_{k,t}/V_{k,t}}{\frac{\sum_{s=t-z}^{t-1} I_{k,s}/V_{k,s}}{z-1}}$$

where $I_{k,t}$ is the implied volatility of security k at time t, $V_{k,t}$ is the volatility of security k obtained from the multi-factor risk model before adjustment at time t and z is the number of past periods over which we choose to observe the relation between implied and multiple factor estimates of volatility.

If we construct orthogonal factors, then the portfolio variance can be written as

$$V_p = \sum_{i=1}^{n}\sum_{j=1}^{n} \sigma_{f(i)}\sigma_{f(j)}\rho_{i,j} + \sum_{i=1}^{m} w_k^2 \sigma_{s(k)}^2$$

$$V_k^* = M_k \times (\sum_{i=1}^{n} \beta_{k,i}^2 \sigma_{f(i)}^2 + \sigma_{s(k)}^2)$$

The key idea is to solve a set of simultaneous equations

$$V_k^*/M_k - \sigma_{s(k)}^2 = \sum_{i=1}^{n} \beta_{k,i}^2 \sigma_{f(j)}^2$$

where

$$\sigma_{s(k,t)} = \max(\sigma_{s(k,t)}, \sigma_{s(k,t-1)} \times p), 0 < p < 1$$

Takeaway: The paper gives a systematic way of incorporating implied volatility in to portfolio risk estimation procedure. This procedure captures the underlying volatility of the individual constituents far better than historical volatility estimates or GARCH processes.